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Hadron Structure and the QCD Phase Transition[1]

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Abstract

Firstly, I give a brief summary of the current understanding of QCD below and near T_c (the critical temperature of the chiral transition). Some emphases are put on the qualitative difference between the Yukawa regime ($T \sim 0$) and the Hagedorn regime ($T \sim T_c$). Secondly, the dynamical phenomena associated with the chiral transition, in particular, the spectral changes of hadrons in hot and/or dense medium are reviewed from the point of view of the QCD spectral sum rules. Confusions on the QCD sum rules at finite temperature/density are also clarified and remarks on the effective lagrangian approaches are given. Thirdly, planned experiments to detect the spectral changes in medium are summarized.

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1 Introduction

Due to the asymptotic freedom, the QCD coupling constant $g(\mu)$ decreases logarithmically as one increases the renormalization scale μ,

$$\frac{g^2(\mu)}{4\pi} \to \frac{12\pi}{(33-2N_f)\ln(\mu^2/\Lambda^2)} \quad . \tag{1}$$

μ is chosen to be a typical scale of the system to suppress the higher order terms in g. Thus, in extremely hot and/or dense QCD medium, μ should be proportional to T (temperature) or the chemical potential, which means that systems at high T and/or high baryon density ρ are composed of weakly interacting quarks and gluons (quark-gluon plasma phase) [1]. At low $T-\rho$ on the contrary, quarks and gluons are confined inside mesons and baryons (hadronic phase). Therefore one may expect a phase transition between the two phases at intermediate T and ρ. The existence of such transition at finite T is in fact "proved" by the numerical simulations of QCD formulated on the lattice [2]. RHIC at BNL and LHC at CERN will serve as machines to create and detect the hot quark-gluon plasma through the relativistic heavy-ion collisions [3]. Several experiments are also planned to detect the partial restoration of chiral symmetry in heavy nuclei through the reactions such as $\gamma + A \to A^* + e^+e^-$ and $p + A \to A^* + e^+e^-$ [4].

The fundamental questions here may be summarized as
(i) What are the properties of the quark gluon plasma?
(ii) What is the precise nature of the phase transition?
(iii) What sort of critical phenomena occurs ?

In the following, I will discuss some of the recent topics related to the above questions with main emphases on (iii). In Section 2, an intuitive view of the QCD phase transition based on the percolation picture is given. In Section 3, the natures of the deconfinement and chiral transition studied on the lattice are briefly summarized. In Section 4, general introduction to the dynamical critical phenomena associated with the chiral transition is given. In section 5, Spectral change of the vector mesons at finite T is examined as a typical and observable example of the dynamical critical phenomena. In Section 6, possibility of the partial restoration of chiral symmetry in heavy nuclei and associated phenomena (such as the mass shift of the vector mesons) are discussed. In Section 7, the planned experiments for detecting the spectral change of the vector mesons are summarized. Section 8 is devoted to concluding remarks.

2 QCD phase transition and continuum percolation

2.1 Transition from Yukawa regime to Hagedorn regime

Let us first start with an intuitive picture of the QCD phase transition given in Fig.1. Imagine heating up the QCD vacuum. At low T, pions (the lightest mode in

Figure 1: A schematic figure of the percolation transition in QCD at $T \neq 0$.

QCD) are thermally excited. As one increases T, massive hadrons are also excited. Since the pions and other hadrons have their own size (e.g. the pion radius is about 0.65fm), the thermal hadrons start to overlap with each other at certain temperature T_c and dissolve into a gas of quarks and gluons above T_c.

Now, what kind of hadrons are mainly populated near the critical temperature? To get a rough idea, let's look at n/T^3 (number density of hadrons divided by T^3) as a function of T given in Fig. 2. The interactions between hadrons are neglected there [5]. The dashed line in Fig. 2 is a contribution of hypothetical massless pions. r denotes the sum of massive resonance contributions other than $\pi, K, \eta, \rho, \omega$. Although only pions are excited below 100 MeV, the resonance contributions start to dominate over pions above 160 MeV. Why this happens? The reason is that the state density $\rho(m)$ of the excited hadrons with mass m increases exponentially as m increases:

$$\begin{aligned} n_{tot}(T) &= \int_0^\infty dm\ n(m;T)\ \rho(m), \\ & \qquad n(m;T) \propto e^{-m/T}, \qquad \rho(m) = \frac{C}{m^\alpha} e^{m/\tau}, \end{aligned} \tag{2}$$

where $n_{tot}(T)$ is the total number density of hadrons, $n(m;T)$ is the number density of a resonance with mass m, and $\alpha = 2$ (2.5), $\tau \simeq 200$ MeV (140MeV) for statistical bootstrap model (for string model). The Boltzman suppression factor $\exp(-m/T)$ in $n(m;T)$ is compensated by the exponentially growing factor $\exp(m/\tau)$ in $\rho(m)$ and one is not allowed to neglect the massive states any more at high T [6]. This is a unique feature of the confining theory like QCD.

I will call the pion-dominated region ($T < 150$ MeV) as **Yukawa regime** and the resonance-dominated region ($T > 150$ MeV) as **Hegedorn regime** because of the obvious reason. Whether T_c is in the Yukawa regime or in the Hegedorn regime is a dynamical question which is examined in the next subsection.

Figure 2: (Number density of hadrons)$/T^3$ as a function of T. r denotes the sum of the resonance contributions other than $\pi, K, \eta, \rho, \omega$ [5].

2.2 Continuum percolation and T_c

Fig. 1 together with Fig. 2 indicates that we may be able to use the knowledge of the so-called continuum or random percolation problem [7] to estimate T_c. The continuum percolation problem is defined as follows. Let us prepare N set of spheres with the same radius R. Then distribute them randomly in a three dimensional box with volume V. If the two spheres touch or overlap with each other, they are said to be connected and form a part of a cluster. Then N and V are taken to infinity with $n = N/V$ being fixed. The problem is to find a critical density n_c at which a cluster with infinite size is first formed. If one replaces the spheres here by hadrons and R by a typical hadron-radius, the close relation between the continuum percolation problem and Fig. 1 becomes clear: n_c can be interpreted as a critical hadron density at which color conductivity starts to be non-vanishing. One can also introduce another critical density where the hadrons are close-packed. This corresponds to an end of the percolation where the full color conductivity is realized.

The continuum percolation problem has been studied using numerical simulations and the renormalization group techniques [8]. The first left column in Table 1 shows the result of the numerical simulations of the onset density of percolation n_c (multiplied by the size of the sphere $v \equiv (4\pi/3)R^3$). The close-packed density is, on the other hand, simply defined by $n_c v = 1$. In the second column, n_c itself is shown by adopting the typical hadronic size as $R = 0.65$fm (which is the pion size). In the third column, corresponding mean distance between the hadrons are shown. When the percolation starts, the mean distance d_c is 3 times larger than the hadron radius, thus the system is still dilute.

Table 1. Critical values for the continuum percolation in QCD.

Figure 3: Possible QCD phase diagram. The precise shape of the critical line and the order of the transition are not known.

	percolation-start (infinite cluster formed)	percolation-end (close-packed)
$n_c v$	0.35 ± 0.06	1.0
$n_c(R = 0.65\text{fm})$	$0.3/\text{fm}^3$	$0.89/\text{fm}^3$
d_c	1.84 fm	1.3 fm

Using Fig. 2 and n_c in Table 1, one can estimate the critical temperature corresponding to the start (end) of the percolation, which results in $T_c \simeq 160$ (180) MeV. In this region, massive resonances dominate over the pions, thus the percolation transition from the hadronic phase to the quark-gluon plasma is likely to occur in the Hagedorn regime. One should also note that, in spite of the large difference between n_c for start and end of the percolation, T_c of the two are very close. This is simply because the resonance contribution to n grows very fast as T increases in the Hagedorn regime as can be seen in Fig. 2. One should note here that the estimate of T_c above is only qualitative: Inclusion of more massive resonances will decrease T_c further.

One can also play similar game for the system at zero T but at high baryon density. In this case, the uncertainty of the critical baryon density ρ_c is large but one gets $\rho_c \simeq$(a few $\sim$ 10)ρ_0 with $\rho_0 = 0.17/\text{fm}^3$ being the normal nuclear-matter density.

On the basis of these intuitive and simple analyses, one may draw a possible phase diagram of QCD (Fig.3). The early universe corresponds to the high T but low ρ region in the phase diagram. Roughly 10^{-5} seconds after the big bang, the universe cools down to about $T \sim 100$ MeV and undergoes a phase transition to the hadronic phase. Deep inside the neutron stars and in the possible quark stars, low T but high ρ matter could be formed due to the strong gravitational pressure. One may also reach the high T and/or high ρ region in the laboratories by using the relativistic heavy-ion colliders such as RHIC and LHC.

Despite the considerable efforts so far, high ρ region is not well understood because of the complex QCD/hadron dynamics. On the other hand, finite T phase

transition is studied rather well since one can make use of the lattice QCD simulations as a useful guide. In the next section, I will briefly summarize the basic knowledge obtained on the lattice for the finite T phase transition.

3 Phase transition on the lattice

The precise determination of the QCD phase transition near T_c is one of the central issues of the recent lattice QCD studies [2]. As for the pure gauge system without dynamical fermions ($m_q = \infty$), the center symmetry ($Z(3)$ in $SU_c(3)$ case) controls the confinement-deconfinement phase transition. The effective $Z(3)$ spin model predicts the 1st order transition and the lattice studies with finite size scaling analyses support this feature [9]. Although it is of 1st order, the transition is much weaker than that seen before on the smaller lattices. Once one introduces dynamical fermions, $Z(3)$ symmetry is explicitly broken. However, as far as m_q is large enough, one can still study the phase transition based on this approximate symmetry. On the other hand, in the opposite limit where m_q is zero, chiral symmetry instead of $Z(3)$ symmetry takes place and $\langle \bar{q}q \rangle_T$ becomes an order parameter. For finite quark masses ($m_{u,d} = O(10\text{MeV})$ and $m_s = O(200\text{MeV})$), chiral symmetry is explicitly broken, but one can study the phase transition based on this approximate chiral symmetry.

The order of the chiral transition for the realistic quark masses are still not known, although the chiral transition near $m_q = 0$ ($q = u, d, s$) is likely to be of 1st order. In the Columbia data [10] with stagerred fermions, the chiral transition is not observed for the realistic values of $m_{u,d,s}$, while the recent Tsukuba data with Wilson fermions [11] indicate that the transition is of 1st order for the realistic values of $m_{u,d,s}$. Future large scale simulations with finite size scaling analyses are called for to settle the issue.

The order of the finite T chiral transition is most relevant to the big-bang nucleosynthesis of ^{9}Be, ^{10}B and ^{11}B [12]. The spatial inhomogeneity due to the bubble formation during the 1st order chiral transition can create those relatively heavy elements, while the standard homogeneous model creates only 10-100 smaller abundances.

From the point of view of the laboratory experiments such as the relativistic heavy-ion collisions, the precise order of the transition is not much relevant because the system size is finite. Instead, the global behavior of entropy density and chiral condensate as a function of T is rather important for the time-evolution of the system as well as the related experimental signals of the formation of the quark-gluon plasma [13].

3.1 Energy density, pressure and chiral condensate

In Fig. 4(a), the lattice data of the energy density and pressure of QCD with 2 light flavors are shown as a function of $\beta = 6/g^2$ [14]. One can clearly see a rapid growth of the energy density $\mathcal{E}$ (square in the figure), which indicates the liberation of quarks

Figure 4: (a) Lattice data for the energy density and pressure in 2-flavors ($ma = 0.025$) on $8^3 \times 4$ lattice [14]. (b) Lattice data for the chiral condensate in (2+1)-flavors on $12^3 \times 6$ lattice (taken from [17] and compiled by W. Weise).

and gluons at high T. $\mathcal{E}$ and entropy density are known to have rapid growth in a narrow range of temperature ($\sim$ 10 MeV) by various numerical simulations on the lattice [15]. On the other hand, the pressure $\mathcal{P}$ (cross in the figure) does not have rapid growth and does not satisfy the Stefan-Boltman's law $\mathcal{E} = 3\mathcal{P}$ above T_c. This may indicate that there still remain strong and non-perturbative interactions between quarks and gluons above T_c. Such non-perturbative effect may have close connection with the confining nature of the dimensionally reduced effective theory of QCD at high T and also with the non-perturbative hadronic correlations above T_c. I will not go into the details of this interesting subject due to the limitation of space (see [16] and the references therein for details).

In Fig. 4(b), the lattice data for the chiral condensates as a function of β in (2+1)-flavors are shown [17]. One can see that (i) a rapid change of the light quark condensate around T_c and (ii) a slow change of the strange condensate across T_c. Whether they have discontinuity across T_c or not is a controversial matter as I have already mentioned.

3.2 Effective theory interpretation

As for the behavior of the chiral condensate shown in Fig.4(b), one can interpret the results using the effective theories of QCD. In Fig. 5, calculations based on the Nambu-Jona-Lasinio (NJL) model [18] and on the chiral perturbation theory (ChPT) + massive resonances [5] are shown. In both cases, a rapid change of the light quark condensate is seen. In the NJL model, the change is driven by the melting of the constituent quark mass. On the other hand, in the ChPT + massive resonances, the massive states turn out to be more important than the pions for chiral restoration, which can be seen by comparing the dash-dotted line (pions) and the shaded area (massive states + pions).

The light quark condensate from the massive resonance contributions is written

Figure 5: Quark condensates in the NJL model [18] (left) and the chiral perturbation theory + massive resonances [5] (right).

as

$$\frac{\langle \bar{q}q \rangle_{T}}{\langle \bar{q}q \rangle_0} = 1 - \int_0^{\infty} dm \, \frac{\langle \bar{q}q \rangle_m}{\langle -\bar{q}q \rangle_0} \, n(m;T) \, \rho(m), \tag{3}$$

where $\langle \bar{q}q \rangle_m$ denotes the expectation value of $\bar{q}q$ with respect to a resonance with mass m. The formula is relevant near T_c and, as in the case of eq.(2), the Boltzman suppression by $n(m;T)$ is compensated by the exponentially growing degeneracy factor $\rho(m)$. At very low temperature, however, the decrease of the condensate is dominated by the pions, namely

$$\frac{\langle \bar{q}q \rangle_{T}}{\langle \bar{q}q \rangle_0} = 1 - \frac{T^2}{8f_\pi^2} - \frac{T^4}{384 f_\pi^4} - \cdots \quad , \tag{4}$$

for massless pion [5]. Fig.5 shows that the critical temperature for the chiral transition is likely in the Hagedorn regime, which is consistent with the argument in section 2 based on the percolation theory.

At present there is no obvious connection between the NJL picture (melting of the constituent mass) and the resonance picture (many massive resonances) in the description of the chiral transition. It is of great interest to find a unified picture of the two. The quark-hadron duality, which can be seen through the dispersion relation in some processes, may be relevant to this problem.

4 Dynamical critical phenomena

Since $\langle \bar{q}q \rangle_{T}$ is a scale dependent quantity, it is not a direct experimental observable. Therefore, one has to look for other physical quantities to see the signal of the chiral phase transition in the real experiments. The spectral change of hadrons at finite T such as the mass shift is one of the possible candidates. In fact, light-hadron masses are essentially determined by the quark condensates as QCD sum rules tell us [19], which suggests that the mass shift of hadrons in medium could be a good measure of the partial restoration of chiral symmetry at finite T and ρ [20, 21, 22].

There actually exist similar situations in condensed matter physics: the existence of the soft phonon modes is an indication that the ground state undergoes structural phase transition and one can study the precise nature of the phase transition by the soft mode spectroscopy (see subsection 4.3 for the examples). In QCD, scalar mesons (fluctuation of the order parameter) and the vector mesons (such as ρ, ω and ϕ) are the candidates for the "soft modes". In particular, if the vector mesons are the soft modes, one can directly detect the spectral changes through the leptonic decays ($\rho, \omega, \phi \to e^+e^-, \mu^+\mu^-$).[3]

Throughout the following sections, I will define the mass of the hadrons as a peak position of a resonance in the spectral function. Before discussing the details of the dynamical critical phenomena, let us first summarize what is know about the static critical phenomena in the next subsection.

4.1 Static fluctuation of the order parameter

The left two figures in Fig. 6 show schematic illustration of the behavior of the chiral condensate $\langle \bar{q}q \rangle$ and its static fluctuation

$$\chi \sim \langle (\bar{q}q)^2 \rangle - \langle \bar{q}q \rangle^2, \tag{5}$$

for QCD with massless two-flavors. (In this case the phase transition is expected to be of 2nd order. Even for the realistic (2+1)-flavors, the similar argument holds as far as the chiral condensate has significant variation near T_c.) Precisely at the temperature where $\langle \bar{q}q \rangle$ vanishes, χ is expected to diverge. The right two figures of Fig. 6 are the corresponding lattice data [24, 25]. "disc" in the lower right figure corresponds to χ which clearly shows a large enhancement near T_c for light quark ($ma = 0.02$). This confirms our theoretical expectation. On larger lattices, one will be able to extract the static critical exponents accurately [26].

Although these static quantities are by themselves interesting, our main concern here is the quantity which can be measured in the laboratories. I will thus concentrate on the time-dependent (dynamical) phenomena in the following sections.

4.2 Dynamical fluctuations and para-pion

A possible dynamical mode which has similar critical behavior with the condensate is the time-dependent fluctuation of the order parameters dictated by the retarded correlation function

$$\Pi(\omega, \mathbf{q}) = i \int d^4x \; e^{iqx} \theta(x^0) \langle [\bar{q}\Gamma q(x), \bar{q}\Gamma q(0)] \rangle_T, \tag{6}$$

where Γ denotes 1 or $i\gamma_5\tau^a$. Expected behavior of the "mass" (peak position of the spectral function $\mathrm{Im}\Pi(\omega, \mathbf{q})$) is shown in the left hand side of Fig. 7. The scalar mode ($\Gamma = 1$) corresponds to σ and the pseudo-scalar mode ($\Gamma = i\gamma_5\tau^a$) corresponds to π.

[3] The spectrum of heavy vector mesons such as J/ψ is also interesting in relation to the physics of confinement and the signature of the formation of the quark-gluon plasma [23]. However, we will not discuss it in this article.

Figure 6: Left two figures: Theoretical expectation for the chiral condensate and the static susceptibility in the 2nd order chiral transition. Right two figures: Corresponding lattice data (upper: $N_f = 2$, $8^3 \times 4$ lattice with $ma = 0.025$ [24], lower: $N_f = 2$, $8^3 \times 4$ lattice with $ma = 0.02, 0.0375, 0.075$ [25].)

The right figure is a calculation using the NJL model with $m_{u,d}(1\mathrm{GeV}) = 5.5\mathrm{MeV}$ [27].

From Fig.7, one confirms the naive expectation that the chiral multiplet will degenerate at and above T_c and also one can learn more:

(i) There is a sizable softening (decreasing mass) of σ below T_c. This is an indication that the ground state is soft for the deformation to the direction of the order parameter. One can also calculate the decay width $\sigma \to 2\pi$ at finite T. Just because of the tendency $m_\sigma \to m_\pi$, the decay width is suppressed near T_c [27]. However, whether σ becomes really a sharp resonance or not depends on the magnitude of the collisional width which has not been calculated for σ so far.

(ii) Above T_c, σ and π are degenerate and have low mass. Furthermore, the decay channel $(\sigma, \pi) \to \bar{q}q$ near T_c is suppressed by the small phase space, therefore the width of the modes are small. In Fig. 8, shown is the spectral function of this degenerate mode above T_c calculated by using the NJL model [21]. We will call this mode as **para-pion** since it can be regarded as a low-mass and long-lived elementary excitation in the para-phase of chiral symmetry.

4.3 Soft modes – examples in solid state physics –

The softening associated with the ferro-para phase transition is well known in sold state physics and the dynamical excitations softened near the critical point are

Figure 7: Left: Theoretical expectation for the masses of the dynamical modes σ and π. Right: Calculation using the NJL model with $N_f = 2$ and $m_{u,d}(1\text{GeV}) = 5.5MeV$ [27].

Figure 8: Spectral function for para-pion above T_c in the NJL model [21].

generally called **soft modes** [28]. In Fig. 9, one of such examples is shown. The figure shows a soft mode in the ferro-electric crystal where the self-polarization $\vec{P}$ is an order parameter [29]. Similar soft mode can be also seen in the ferro-elastic crystal where the self-distortion $\vec{X}$ is an order parameter [30].

5 Vector mesons at finite T

Although the softening of the scalar meson σ below T_c and the existence of para-pion above T_c are the interesting dynamical critical phenomena, it is rather difficult to measure them in the relativistic heavy ion collisions. The reason is that the hadronic decay of σ is masked by thousands of pions produced by the collisions. There might be a chance, however, to see the decay $\sigma \to 2\gamma$ which occurs when the system is at

Figure 9: Soft mode in the ferro-electric crystal [29].

finite density [31].

On the other hand, the light vector mesons such as ρ, ω and ϕ are more interesting from the experimental point of view. They decay into lepton pairs (e^+e^- and $\mu^+\mu^-$) which can penetrate the hot/dense medium without strong interactions. Thus, if vector mesons are the soft modes (which is not obvious from the outset), the lepton pairs are the good probe to see the dynamical critical phenomena. To study this possibility, let's start with the retarded hadronic correlation at finite T in the vector channel:

$$\Pi^V_{\mu\nu}(\omega, \mathbf{q}) = i \int d^4x \ e^{iqx} \theta(x^0) \langle [J^V_\mu(x), J^V_\nu(0)] \rangle_{T}, \tag{7}$$

where $J^{\rho,\omega}_\mu = \bar{u}\gamma_\mu u \mp \bar{d}\gamma_\mu d$ and $J^\phi_\mu = \bar{s}\gamma_\mu s$. $\Pi^V(\omega) \equiv \Pi^V_{\mu\mu}(\omega, \mathbf{q}=0)/(-3\omega^2)$ satisfies the following dispersion relation

$$\Pi^V(\omega) = \frac{1}{\pi} \int_0^\infty \frac{\mathrm{Im}\Pi^V(\omega')}{\omega'^2 - (\omega + i\epsilon)^2} d\omega'^2 + (\text{subtraction}). \tag{8}$$

What we are interested in is the spectral change of $\mathrm{Im}\Pi^V$ at finite T.

5.1 Low T theorem in the Yukawa regime

At extremely low temperature, spectral changes of hadrons are controlled solely by chiral symmetry. In fact, the forward scattering of thermal pions by a hadron is only the relevant process to change the properties of the hadron at low T. Leutwyler and Smilga have shown that the masses of light hadrons (nucleon, ρ-meson etc) do not change at $O(T^2)$, though the pole residues can be modified at this order [32].

For the ρ meson, the spectral change at $O(T^2)$ can be easily calculated by using the soft pion theorem [33];

$$\mathrm{Im}\Pi^V(\omega) \simeq (1 - \frac{T^2}{6f_\pi^2}) \mathrm{Im}\Pi^V_{T=0}(\omega) + \frac{T^2}{6f_\pi^2} \mathrm{Im}\Pi^A_{T=0}(\omega), \tag{9}$$

Figure 10: (a) Mixing between ρ and a_1 mesons through thermal pion. (b) Mixing between ρ and other hadrons through thermal resonances.

where $\Pi^{V,A}_{T=0}(\omega)$ denotes the correlation function at zero T in the vector ($V = \rho$-meson) and axial-vector ($A = a_1$-meson) channel. It is obvious from the formula that the thermal pions induce the mixing between V and A channels and also modify the pole residues, but do not change the mass. See Fig.10(a) for the physical process to induce the V-A mixing.

5.2 Approaching T_c – Hagedorn regime –

As we have seen in Section 2, the real interesting region is in the Hegedorn regime where resonances dominate over pions. In this regime, the vector mesons will have interactions with various thermal resonances and the final states are not limited to the axial vector meson. See Fig. 10(b). In this complex situation, there are two possible ways to study the spectral change.

(i) Lattice QCD: By measuring the imaginary time correlation on the lattice, one can in principle reconstruct $\mathrm{Im}\Pi^V(\omega)$ through the dispersion relation [34]

$$\Pi^V(i\omega_n) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi^V(\omega')}{\omega'^2+\omega_n^2}d\omega'^2 + (\text{subtraction}), \tag{10}$$

where $\omega_n = 2n\pi T$ is the Matsubara frequency.

(ii) QCD sum rules: By calculating the real time correlation in the deep Euclidian region using the operator product expansion, one can in principle reconstruct $\mathrm{Im}\Pi^V(\omega)$ through the dispersion relation [35, 36]

$$\mathrm{Re}\Pi^V(\omega^2 \to -\infty) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi^V(\omega')}{\omega'^2-\omega^2}d\omega'^2 + (\text{subtraction}). \tag{11}$$

The lattice approach is still far from being realistic because the temporal lattice size is currently too small. We will thus pursue the QCD sum rules in the following to get some constraints on the spectral function.

One should note here that the hadronic screening mass defined by the "spatial" hadronic correlation does not have direct connection with the "real mass" which we are working on here. The calculation of the screening mass on the lattice is, however, much easier than the real mass and it has been and is being studied extensively [37].

5.3 QCD sum rules at finite T

QCD sum rules in medium [36] start with the following operator product expansion (OPE) for $\Pi^V(Q^2)$ with $Q^2 \equiv -\omega^2$;

$$\mathrm{Re}\Pi^V(Q^2) = -C_0 \ln Q^2 + \sum_{n=1}^{\infty} \frac{C_n}{Q^{2n}} \langle \mathcal{O}_n \rangle_T, \tag{12}$$

where C_n are the c-number Wilson coefficients which are T independent. All the medium effects are in the thermal average of the local operators $\mathcal{O}_n$. Since $\langle \mathcal{O}_n \rangle_T \sim T^{2l} \cdot \Lambda_{QCD}^{2m}$ with $l+m=n$ due to the dimensional reason, (12) is a valid asymptotic expansion as far as $Q^2 \gg T^2, \Lambda_{QCD}^2$.

The first 4-terms of $C_n \langle \mathcal{O}_n \rangle_T$ has been calculated as [36]

$$C_0 = -\frac{1}{8\pi}(1+\frac{\alpha_s}{\pi}), \quad C_1 = 0, \tag{13}$$

$$C_2 \langle \mathcal{O}_2 \rangle_T = \frac{1}{24} \langle \frac{\alpha_s}{\pi} G^2 \rangle_T + \frac{4}{3} \langle \mathcal{S} \bar{q} i \gamma_0 D_0 q \rangle_T, \tag{14}$$

$$C_3 \langle \mathcal{O}_3 \rangle_T = -\langle \mathrm{scalar\ 4-quark} \rangle\rangle_T + \frac{16}{3} \langle \mathcal{S} \bar{q} i \gamma_0 D_0 D_0 D_0 q \rangle_T. \tag{15}$$

Here we have neglected the terms proportional to the light quark masses and the quark-gluon mixed operators. Also, $\mathcal{S}$ makes the operators symmetric and traceless. At low T (Yukawa regime), one may use the soft pion theorems and the parton distribution of the pion to estimate the r.h.s. of the above equations. When T is close to T_c, one has to look for totally different way of estimation: a simplest approach is to assume the resonance gas to evaluate the r.h.s., while the direct lattice simulations will be the most reliable way in the future.

An important feature of the OPE in the above is that local operators with Lorentz indices arise. This happens because we are taking the rest frame of the heat bath which breaks covariance. To see the effect of the operators with Lorentz indices, let's look at the T dependence of the r.h.s. of (14) using the resonance gas approximation of π, K and η. As is seen from Fig.11, these resonances increase the second term of the r.h.s. of (14) considerably, which acts to reduce the mass of the vector mesons as we will see later.

Using OPE given in (12) and the dispersion relation (11), one can construct the following energy weighted sum rules or the finite energy sum rules (FESR) [36];

$$I_1 = \int_0^{\infty} [\mathrm{Im}\Pi^V(\omega) - \mathrm{Im}\Pi^V_{pert.}(\omega)] d\omega^2 = 0, \tag{16}$$

$$I_2 = \int_0^{\infty} [\mathrm{Im}\Pi^V(\omega) - \mathrm{Im}\Pi^V_{pert.}(\omega)] \omega^2 d\omega^2 = -C_2 \langle \mathcal{O}_2 \rangle_T, \tag{17}$$

$$I_3 = \int_0^{\infty} [\mathrm{Im}\Pi^V(\omega) - \mathrm{Im}\Pi^V_{pert.}(\omega)] \omega^4 d\omega^2 = C_3 \langle \mathcal{O}_3 \rangle_T. \tag{18}$$

Here $\mathrm{Im}\Pi^V_{pert.}(\omega)$, which is T independent, denotes the imaginary part corresponding to the perturbative part of Π^V. Similar sum rules hold for the axial vector channel (in the chiral limit) except that one has a different operator for $\mathcal{O}_3$. One can also generalize the above sum rules to finite $\mathbf{q}$ [38].

Figure 11: $C_2\langle\mathcal{O}_2\rangle_T$ with $(a+b+c)$ and without $(a+b)$ the contribution $\langle\bar{q}i\gamma_0 D_0 q\rangle_T$ in the resonance approximation [36]. The solid (dashed) line includes the thermal contribution of π (π, K, η).

We have only three constraints $I_{1,2,3}$ from the QCD sum rules as shown above. This is because we could calculate only first three non-perturbative terms $C_1\mathcal{O}_1$, $C_2\mathcal{O}_2$ and $C_3\mathcal{O}_3$ in OPE. By the three constraints, we can determine three resonance parameters but not more than three. In this situation, the most economical way to parametrize the spectral function $\mathrm{Im}\Pi^V(\omega)$ is to introduce the following three parameters; the position of the resonance peak $m(T)$, the continuum threshold $S_0(T)$ and the area integral of the resonance $F(T) = \int_{0+}^{S_0} \mathrm{Im}\Pi^V(\omega) d\omega^2$.[4]

With three sum rules and the parametrization of $\mathrm{Im}\Pi^V(\omega)$ in the above, it is impossible to extract the width of the resonance. A honest way to determine the width is to go to dim. 8 operator $C_4\mathcal{O}_4$ and derive one more sum rule, which is practically a formidable task.

5.4 Spectral change in Yukawa regime and Hagedorn regime

Yukawa regime:

By examining the FESR or the Borel sum rules at low T, one can show the followings.

(i) The low T theorem (9) is satisfied [36, 39];[5]

$$\delta m_{V,A} = 0 \quad \text{at} \quad O(T^2). \tag{19}$$

(ii) Negative mass shifts occur at $O(T^4)$ [39];

$$\delta m_V \sim \delta m_A = -cT^4 \quad \text{at} \quad O(T^4), \tag{20}$$

[4] $\mathrm{Im}\Pi^V(\omega)$ has a Landau-damping contribution at $\omega = 0$ which we have calculated explicitly in the resonance gas approximation and is not included in $F(T)$ [35, 36].

[5] The difference of the OPE in vector channel and the axial-vector channel appears only in the dim.6 operator [36]. At low T, this difference is completely absorbed by the $V-A$ mixing and does not cause mass shift, which can be seen explicitly [39] or numerically [36] in QCD sum rules. The similar situation also arises for the nucleon mass at $O(T^2)$ [40].

Figure 12: Temperature dependence of the ρ, ω mass and the continuum threshold S_0 in the Borel sum rule in CASE-II. The upper (lower) curves for $m_{\rho,\omega}$ and S_0 correspond to the case (a) ((b)) in Table 2.

with a small positive coefficient c.

Hagedorn regime:

When T is close to T_c, the sum rules are not powerful enough to predict the precise spectral change. In fact, there are two physical possibilities.

CASE-I: The widths of the hadrons increase rapidly and no distinction between the resonance and the continuum can be made near T_c.

CASE-II: There still have clear distinction between the continuum and the lowest resonance near T_c.

Since CASE-II is a more interesting possibility experimentally and also CASE-II is actually realized in many solid state examples, let's focus our attention on it for the moment and try to see what kind of constraint one can make from the QCD sum rules. In Fig.12, the mass and the continuum threshold for the ρ, ω mesons at finite T are shown. The calculation was made using the Borel version of the QCD sum rules (FESR gives essentially the same results). The curves (a) and (b) correspond to the assumptions (a) and (b) in Table 2 respectively.

The decreasing mass and threshold are clearly induced by the dimension 4 operators, and if dimension 6 operator decreases at finite T as (b), it enhances the decrease further. Note here that $\langle \mathcal{O}_3 \rangle_T$ in (b) is simply an ansatz: near $T=0$, more rigorous T dependence is know for this quantity [36].

Table 2. Assumed dimension 4 and 6 condensates in Fig. 12.

	$\langle \mathcal{O}_2 \rangle_T$	$\langle \mathcal{O}_3 \rangle_T$
(a)	π, K, η resonance gas	no T-dependence
(b)	π, K, η resonance gas	mean-field ansatz $\sim (1 - T^2/T_c^2)^{1/2}$

Figure 13: ϕ-meson mass as a function of T in the QCD sum rules with resonance gas approximation [42].

5.5 ϕ meson near T_c

As we have seen in Fig.4(b), the strangeness condensate decreases more slowly than the u-d condensate at finite T. Nevertheless, one could probe the decrease through the spectral change of the ϕ meson: Since ϕ width is rather small in the vacuum (4.5 MeV) and will not increase more than 25 MeV even at $T \simeq 180 MeV$ [41], the experimental uncertainty for detecting the spectral change of ϕ will be less than that for ρ. Asakawa and Ko have generalized the method of [36] and calculated the ϕ mass at finite T in CASE-II in the resonance gas approximation of $\pi, \rho, \omega, K, K^*, \eta, N, \Delta, \Lambda, \Sigma$ [42]. As is shown in Fig. 13, ϕ mass decreases, which is mainly induced by the decrease of the dimension 4 condensate $m_s \langle \bar{s}s \rangle_T$. The observable consequence of this shift is the double ϕ peak in the e^+e^- spectrum at RHIC and LHC which I will come back in section 7.

5.6 Collision widths of vector mesons

Since QCD sum rules do not tell us the change of the hadronic widths, it will be desirable to see how large the widths could be at finite T using other method. Haglin has recently evaluated the collisional widths of ρ, ω and ϕ through the process

$$r + V \to (\mathrm{two-body\ final\ states}), \tag{21}$$

where V denotes the vector mesons and r is the thermally excited resonances ($\rho, \omega, \pi, K, K^*, \phi$ are taken into account in the calculation) [41].

Fig. 14 shows that ρ and ω acquire 50-100 MeV width near $T = 180$ MeV, while ϕ acquires at most 25 MeV width. These widths are comparable or even larger than the widths in the vacuum ($\Gamma_\rho = 150$ MeV, $\Gamma_\omega = 8.5$ MeV and $\Gamma_\phi = 4.5$ MeV). Nevertheless, the total width is still small for ω and ϕ and they can be considered as distinct resonances. ρ is marginal in this respect. However, the negative mass shift of ρ (which is not considered and is not attainable in Haglin's kinetic theory approach) has an effect to reduce the width because of the phase space suppression

Figure 14: Collision width of ϕ in the resonance gas of $\rho, \omega, \pi, K, K^*, \phi$ [41].

of $\rho \to 2\pi$. Thus there is still a possibility to have rather distinct ρ if its mass has downward shift.

5.7 T-dependent Wilson coefficients – Are they meaningful ? –

In the early stage of the application of the QCD sum rules in the medium [35, 43] there was a confusion on the OPE at finite temperature: the thermal quark propagator was used to calculate the Wilson coefficients of OPE series, which gives T-dependent Wilson coefficients. Although it was later clarified that such procedure does not make sense [33, 36, 39] and correct sum rules have been developed [36], the confusion is still floating around in the literatures (see e.g. [44]). So I will make a few remarks on this point here.

(i) First of all, one should remember that OPE of $J(x)J(0)$ is based on the factorization of the soft scale and hard scale [45]. The soft dynamics is in the matrix elements of local operators while the hard dynamics is in the Wilson coefficients which are by definition independent of the states sandwiching the current product $J(x)J(0)$ (target independence of the Wilson coefficients). This factorization property is a basis of the whole success of the deep inelastic scattering physics. In our case, the hard scale to be taken into account in the Wilson coefficients is Q and the soft scales to be taken into account in the matrix elements are $T(<250\text{MeV})$ and $\Lambda_{QCD}(\sim 200\text{MeV})$. As fas as one keeps $Q \gg T, \Lambda_{QCD}$, OPE is unique and there is no room to have T-dependent Wilson coefficients.

(ii) It is physically erroneous to use free thermal quark-propagator in calculating the correlation function below T_c. Since quarks are strongly interacting and confined below T_c, there is no reason to believe that the quarks have free thermal distribution even close to T_c.

(iii) The statement (i) in the above has nothing to do with the complete set $\mid l\rangle$ one adopts to evaluate the thermal average of $\sum_l \langle l \mid J(x)J(0) \mid l\rangle \exp(-E_l/T)$ in (7). In other words, eq.(12) is an exact expression when $Q \gg T, \Lambda_{QCD}$. How

to calculate $\langle \mathcal{O}_n \rangle_T$ is a different problem: the direct lattice QCD simulation will become the best way to estimate the magnitude in the future. At present, what one can do at best is to assume resonance gas of hadrons near T_c to estimate $\langle \mathcal{O}_n \rangle_T$.

In conclusion, the answer to the headline of this subsection is NO, and analyses based on the T-dependent Wilson coefficients are not justified.

5.8 Remarks on the effective theory approaches

There exist many attempts so far to calculate the spectral change of the vector mesons using different versions of the gauged non-linear (and linear) σ-models [46]. In most of these models, the basic ingredients in the Lagrangian are limited to the vector mesons and pions (and σ meson). Since the pion is the lightest particle, it is dominantly excited at low temperature and continued to be dominant even at high T since there are no other possible resonances in the lagrangian.

As we have already shown in detail in section 2 (see e.g. Fig.2), the description of the hot matter using pion alone is inadequate for chiral restoration and massive resonance contribution is inevitable to get realistic value of T_c. In this respect, the above model calculations, which shows increasing (decreasing) ρ (a_1) mass and constant ω mass are valid only at low temperature (Yukawa regime) and totally different method is required to predict the mass shift near T_c (Hagedorn regime).

A possible way to incorporate the higher resonance contribution effectively is to use the chiral quark model [47] where vector mesons are interacting with constituent quarks with mass $M \simeq 350$ MeV. In such models, the increasing number of resonances and the simultaneous decrease of the quark condensate at finite T are simulated by the decreasing $M(T)$. Then one can make a physical argument that the vector meson mass decreases as far as $M(0) > M(T)$ [48]. Another effective theory predicting the decreasing vector meson mass is the Brown and Rho's lagrangian [49] based on the Georgi's vector symmetry [50]. The relation of this approach to the previous one is not known however.

6 Partial restoration of chiral symmetry at finite density

In the preceding sections, we have concentrated on the hadron properties at finite T. What we found is that the hadronic mass shift as well as the change of the chiral condensate occur only when T is close to T_c. At finite baryon density, the situation is quite different and one may expect partial restoration of chiral symmetry even in the heavy nuclei. The basis of this assertion is that the quark condensate calculated in the Fermi gas approximation decrease considerably in nuclear matter [51, 52].

$$\frac{\langle \bar{u}u \rangle_\rho}{\langle \bar{u}u \rangle_0} = 1 - \frac{4\Sigma_{\pi N}}{f_\pi^2 m_\pi^2} \int^{p_F} \frac{d^3p}{(2\pi)^3} \frac{m_N}{E_p}, \tag{22}$$

where $E_p \equiv \sqrt{p^2 + m_N^2}$, $\Sigma_{\pi N} = (45 \pm 10)$MeV, and the integration for p should be taken from 0 to the fermi momentum p_F($p_F = 270$ MeV at normal nuclear matter

Figure 15: Condensates vs ρ/ρ_0 in nuclear medium: $S = \langle \bar{s}s \rangle_\rho / \langle \bar{s}s \rangle_0$, $U = \langle \bar{u}u \rangle_\rho / \langle \bar{u}u \rangle_0$ and $G = \langle G^2 \rangle_\rho / \langle G^2 \rangle_0$. As for the OZI violation in the nucleon $y = 2\langle \bar{s}s \rangle_N / \langle \bar{u}u + \bar{d}d \rangle_N = 0.12$ is used. This figure is taken from the third reference in [1].

density $\rho_0 = 0.17/\mathrm{fm}^3$). At $\rho = \rho_0$, the above formula gives $(34\pm 8)\%$ reduction of the chiral condensate from the vacuum value. See Fig.15. The corrections to this simple fermi-gas approximation arise in higher orders in p_F. Several estimates show that such corrections are small at ρ_0 [53]. If this is the case, the heavy nuclei is a good testing ground of the partial chiral restoration: the vector mesons in nuclei will be the best probe to see the effect.

6.1 Vector mesons in nuclear matter

Let's first consider ρ and ω mesons propagating inside the nuclear matter. Adopting the same fermi-gas approximation with (22) and taking the vector meson at rest ($\mathbf{q} = 0$) for simplicity, one can generally write the mass-squared shift (or the self-energy) $\delta m_{_V}^2 \equiv m_{_V}^{*2} - m_{_V}^2$ as

$$\delta m_{_V}^2 = 4 \int^{p_f} \frac{d^3 p}{(2\pi)^3} \frac{m_{_N}}{E_p} f_{VN}(\mathbf{p}), \tag{23}$$

where $f_{VN}(\mathbf{p})$ denotes the vector-meson (V) – nucleon (N) forward scattering amplitude in the relativistic normalization (see Fig.16). Here, we took spin-isospin average for the nucleon states in f_{VN}; that is why we have a degeneracy factor 4 in the r.h.s. of (23). The integration for p should be taken up to the fermi momentum $p_{_F}$.

If one can calculate $f_{VN}(\mathbf{p})$ reasonably well in the range $0 < p < p_{_F} = 270$ MeV (or 1709 MeV $< \sqrt{s} <$ 1726 MeV in terms of the $V - N$ invariant mass), one can predict the mass shift. Unfortunately, this is a formidable task. First of all, $f_{VN}(\mathbf{p})$ is not a constant at all in the above range since there are at least two s-channel resonances $N(1710), N(1720)$ in the interval and two nearby resonances $N(1700)$ and $\Delta(1700)$. They all couple to the $\rho - N$ system [54] and give a rapid variation of $f_{VN}(\mathbf{p})$ as a function of p. Secondly, there are t-channel meson exchanges between

Figure 16: Forward scattering of vector meson with the nucleon in nuclear matter.

ρ and N, which are difficult to estimate since we do not know what are the relevant mesons and what are their couplings with ρ and N. Even if one could manage these s-channel and t-channel contributions assuming some effective lagrangian, the connection of the resulting forward amplitude and the chiral condensate is still missing. Therefore one should look for totally different approach to estimate δm_V, one of which is the QCD sum rules in medium discussed in the following subsections.

6.2 Constraints from QCD sum rules

Starting from the retarded correlation of the vector currents in nuclear medium, one can write down the following FESR constraints [55].

$$\int_0^\infty [\mathrm{Im}\Pi^V(\omega) - \mathrm{Im}^V\Pi_{pert.}(\omega)]\omega^{2(n-1)}d\omega^2 = (-)^{n-1}C_n\langle\mathcal{O}_n\rangle_\rho, \tag{24}$$

with $n = 1, 2, 3$. In the r.h.s., the matrix elements of the local operators are calculated in the fermi gas approximation. If one introduces three parameters as before, namely the peak position of the resonance, the continuum threshold and the integrated strength of the resonance, one can extract the density dependence of these parameters from (24). In Fig. 17, results of such analysis using the Borel sum rule are shown (FESR gives essentially the same result) [55]. By making the linear fit, one could deduce a formula for the mass shift as

$$\frac{m^*_{_V}}{m_{_V}} \simeq 1 - c_{_V}\frac{\rho}{\rho_0}, \tag{25}$$

where $c_{\rho,\omega} = 0.18 \pm 0.05$ and $c_\phi = (0.15 \pm 0.05)y$ with $y = 0.1 - 0.2$ being the OZI breaking parameter in the nucleon (see the figure caption of Fig.15 for the definition). These numbers are obtained by neglecting the contribution of the quark-gluon mixed operator with twist 4; inclusion of them moves the central value of $c_{\rho,\omega}$ to 0.15 [56].

The scaling argument of Brown and Rho [22] and the Walecka model of nuclear matter [57] also predict the similar decrease of the ρ and ω masses.

Figure 17: (a) Masses of ρ, ω and ϕ mesons in nuclear matter predicted in the QCD sum rules (left) [55] together with the prediction of the Walecka model (right) [57]. M^*/M in the right figure shows the effective mass of the nucleon.

6.3 Use and misuse of the QCD sum rules in nuclear medium

Let me comment more on the formula (23), since it has created confusion in the literatures (see e.g. [58]) on the vector mesons in nuclear matter.

As I have mentioned, $f_{VN}(\mathbf{p})$ must be a rapidly varying function of p. Thus it is impossible to approximate it by $f_{VN}(0)$:

$$f_{VN}(\mathbf{p}) \neq f_{VN}(0) \qquad \text{except for} \;\; p \sim 0. \tag{26}$$

Note that the $V-N$ scattering length $a_{_{VN}}$ is proportional to $f_{VN}(0)$.

In terms of the mass shift, the (invalid) approximation $f_{VN}(\mathbf{p}) = f_{VN}(0)$ for $0 < p < p_F$ implies that

$$\delta m_{_V}^2 \simeq f_{VN}(0)\rho. \tag{27}$$

Although this formula is valid at extremely low density, it is *useless* at nuclear matter density due to (26).

Motivated by the formula (27), however, it is claimed in ref.[58] that the mass shift is positive since $f_{VN}(0)$ is positive in a QCD sum rule estimate. It can be shown that this claim is erroneous [56]:

(i) Use of the formula (27) in nuclear matter is wrong from the outset.

(ii) The calculation of f_{VN} in [58] is wrong due to the following reason. The author starts with the QCD sum rules for the scattering amplitude $\Pi^{VN} = \langle N \mid \mathrm{T}J_\mu(x)J_\nu(0) \mid N\rangle$. For $\mathrm{Im}\Pi^{VN}$, <u>three</u> phenomenological parameters are introduced (one of them is $f_{VN}(0)$). On the other hand, only <u>two</u> sum rules can be obtained from the OPE of $\mathrm{Re}\Pi^{VN}$. Now, one cannot solve three unknowns from two equations without using a magic. To get a valid estimate of $f_{VN}(0)$, one needs higher orders in OPE, which is practically very difficult to do for Π^{VN}. Thus it is impossible to get reliable result for the scattering length in QCD sum rules at the present stage.

(iii) The author also claims that, in the low density approximation (which is different from the fermi-gas approximation), (27) can be derived from the medium sum rules for $\omega^2\Pi^V(\omega)$. This statement is true, but it can be easily shown that the sum rules for $\omega^2\Pi^V(\omega)$ have the same problem with (ii) and are useless without the information on dim. 8 operators. Also, one cannot derive (27) from $\Pi^V(\omega)$ even in the low density approximation. Thus the above claim does not have any influence on the validity of the sum rules used in the preceding sections.

Let us summarize again the lessons we learned in this subsection: Firstly, the mass shift and the scattering length does not have direct connection in nuclear matter due to the momentum dependence of the $V-N$ forward scattering amplitude. Secondly, sum rules for the $V-N$ scattering amplitude cannot predict the $V-N$ scattering length without dimension 8 operators in OPE. Thirdly, sum rules for $\omega^2\Pi^V(\omega)$ does not work at all even in the vacuum without dimension 8 operators and so does in the medium. Thus all the claims given in ref.[58] are invalid. Also, only the consistent sum rules in medium currently available is the one starting from $\Pi^V(\omega)$ given in [55].

There is a similar confusion on the nucleon in nuclear matter [59], which is actually predated ref.[58]. For the $N-N$ forward scattering amplitude $f_{NN}(p)$, one can prove that it has a huge p dependence at low p using the low energy $N-N$ phase shift [56]. The deutron resonance in ${}^3S_1$ channel and the strong attraction in ${}^1S_0$ channel near $p=0$ induce a rapid variation of $f_{NN}(p)$, which makes one impossible to approximate the amplitude by $f_{NN}(0)$ in the interval $0 < p < p_{_F}$. Thus, the N-N scattering length and the optical potential for the nucleon in nuclear matter have no connection.

6.4 Remarks on the effective theory approaches

There exist many attempts so far to calculate the spectral change of the vector mesons using effective models. The calculation by Chin [61] using the Walecka model predict the increasing ω-meson mass in medium due to the scattering process

$$\omega + N \to N \to \omega + N. \tag{28}$$

For the ρ-meson, more sophisticated calculations including Δ and in-medium pion contributions predict a slight increase of the ρ-mass [62]. In all these calculations, only the effect of the polarization of the Fermi sea is taken into account.

On the other hand, Kurasawa and Suzuki have stated in clear terms that the mass of the ω-meson is affected substantially by the vacuum polarization in medium

$$\omega \to N_* \bar{N}_* \to \omega, \tag{29}$$

where N_* is the nucleon in nuclear medium [63]. The vacuum polarization dominates over the Fermi-sea polarization and leads decreasing vector meson mass. This conclusion was confirmed later by other authors [64] and also generalized to the ρ-meson [57].

What is missing in the *Fermi sea* approaches [61, 62] is the effect of the scalar mean-field on the vector meson mass. On the other hand, *Dirac sea* approaches

[63, 64, 57] have close similarity with the other mean-field models such as those of Brown and Rho [22], Jaminon and Ripka [65], and Saito and Thomas [66], which predict the decrease of the vector-meson masses. It is desirable to develop a unified effective lagrangian which embodies the essential part of these approaches [48].

7 Planned experiments – What one should look for? –

7.1 finite T case

As I have emphasized several times, the ϕ meson is an excellent candidate to see the dynamical critical phenomena at finite T. Asakawa and Ko have proposed the so-called double ϕ-peak signal which is shown in Fig.18 [42]. Fig. 18(a) shows the time history of the hot region created by the relativistic heavy ion collisions. Central plateau in the figure shows a slow conversion from the quark gluon plasma phase to the hadronic phase. The first order phase transition is not necessary to have this plateau: Rapid (but continuous) change of the entropy in a narrow temperature interval, which is actually seen on the lattice [15], is enough for the long duration of the plateau. ϕ-mesons in the lepton pair spectrum should have two components if the initial temperature is high enough as Fig. 18 (b). One is the shifted ϕ decaying from the plateau region. The other is the ϕ decaying at later stages. Longer the duration time of the plateau, the higher the shifted peak-height grows. Such signals could be seen at RHIC and LHC where the quark-gluon plasma is expected to be created in the initial stage of the collisions.

7.2 finite ρ case

The vector meson mass shift at finite baryon density could be seen in heavy nuclei. There exit already two proposals to look for it [4]. One is by Shimizu et al.: They propose an experiment to create ρ and ω in heavy nuclei using coherent photon - nucleus reaction and subsequently detect the lepton pairs from ρ and ω. Enyo et al. propose to create ϕ meson in heavy nuclei using the proton-nucleus reaction and to measure kaon pairs as well as the lepton pairs. By doing this, one can study not only the mass shift but also the change of the leptonic vs hadronic branching ratio

$$r = \Gamma(\phi \to e^+e^-)/\Gamma(\phi \to K^+K^-), \tag{30}$$

which is sensitive to the change of the ϕ-mass as well as K-mass in medium.

In Table 3, some details about the above planned experiments are summarized. There are also on-going heavy ion experiments at SPS (CERN) and AGS (BNL) where high density matter is likely to be formed. In particular, CERES/NA45 at CERN recently reported an enhancement of the e^+e^- pairs below the ρ resonance, which is hard to be explained by the conventional sources of the lepton pairs [67]. Also, E859 at BNL-AGS reported a possible spectral change of the ϕ-peak in K^+K^- spectrum [68]. If these effects are real, the shoulder structure of the spectrum

Figure 18: (a) Proper-time (τ) history of the hot matter. (b) Double ϕ peak expected in the lepton pair spectrum. The peak around 0.78GeV is the ω which is assumed not to have mass shift. The figures are taken from [42].

expected by the mass shift of the vector mesons could be a possible explanation [69].

Table 3: Planned experiments aiming to detect the spectral change of vector mesons.

8 Concluding remarks

The spectral change of the elementary excitations in medium is an exciting new possibility in QCD. By studying such phenomenon, one can learn the structure of the hadrons and the QCD ground state at finite (T, ρ) simultaneously. Some theoretical models predict that the light vector mesons (ρ, ω and ϕ) are sensitive to the partial restoration of chiral symmetry in hot/dense medium. These mesons are also experimentally good probes since they decay into lepton pairs which penetrate the hadronic medium without loosing much information. Thus, the lepton pair spectroscopy in QCD will tell us a lot about the detailed structure of the hot/dense matter, which is quite similar to the soft-mode spectroscopy by the photon and neutron scattering experiments in solid state physics. The theoretical approaches to study the spectral changes are still in the primitive stage and new methods beyond QCD sum rules and naive effective lagrangian approaches are called for.

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